



## Letter to the Editor

## A note on “Investigation and modelling of the wall pressure field beneath a turbulent boundary layer at low and medium frequencies”

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Received 2 April 2003

In Ref. [1] large volume of experimental investigations of the wall pressure beneath a turbulent boundary layer is presented. These results undoubtedly increase ones knowledge, which is necessary for solving many practical tasks connected with flow-induced noise and vibration problems [2].

The main attention in Ref. [1] is given to the investigation of the cross-spectrum density

$$S_{pp}(\xi_1, \xi_2, \omega) = \phi(\omega) |\gamma(\xi_1, \xi_2, \omega)| e^{i\theta}. \quad (1)$$

Here  $\phi(\omega)$  is the wall pressure power density,  $|\gamma(\xi_1, \xi_2, \omega)|$  and  $\theta$  are the modulus and the phase of a dimensionless cross-spectrum, which characterizes at the frequency  $\omega$  the statistical connection between the pressure fluctuations at the two points on the wall and  $\xi_1, \xi_2$  are the distances between these points streamwise and spanwise, respectively. The results of the measurements [1] are submitted in the form  $\gamma^2$  versus  $\omega\delta/U_\tau$  for different values  $\xi_i/\delta$  ( $i = 1, 2$ ). Here  $\delta$  is the thickness of a boundary layer and  $U_\tau$  is the friction velocity. The experimental data are compared with results of calculations by models of both Efimtsov [3] and Smol'yakov and Tkachenko [4] (Figs. 13 and 14, Ref. [1]).

Unfortunately, the legends for Figs. 13 and 14 contain mistakes: the Efimtsov's model is designated as the Smol'yakov–Tkachenko's model and vice versa. The legend for Fig. 14 says that it is the data for longitudinal separations  $\xi_1$  ( $\xi_2 = 0$ ) and actually in Fig. 14 the data for transversal separations  $\xi_2$  ( $\xi_1 = 0$ ) are shown.

In Fig. 10 results of the phase speed measurements with the Efimtsov's model are compared but the account is incorrect. The correct calculated line is shown in the present Fig. 1. It is calculated using the formulas of Ref. [3] and data from Table 2, Ref. [1]. It is visible that the errors are very large which contradicts the statement of the authors: “Figure 10 shows the phase velocity against frequency at several transducers separation, along with model proposed by Efimtsov.” (p. 494).

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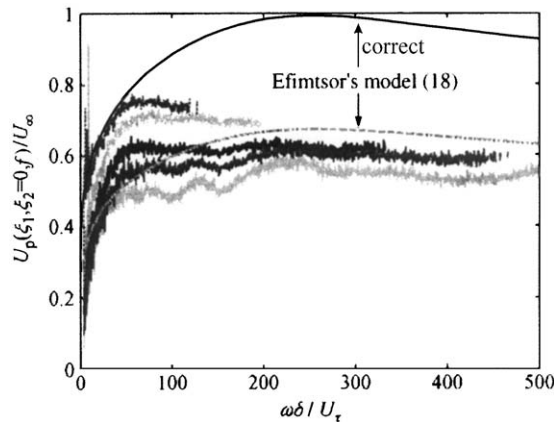


Fig. 1. Streamwise phase velocity for  $U = 40$  m/s,  $U_\tau = 1.47$  m/s and  $\delta = 28.8$  mm solid line—correct calculation, dotted line—wrong calculation [1].

As a result of approximation of the experimental data the authors of Ref. [1] have offered a rather unusual model for the coherence function of wall pressure turbulent fluctuations:

$$|\gamma(\xi_1, \xi_2, \omega)| = \exp\left(-\sqrt{vf^3(\alpha_1^2 \xi_1^2 + \alpha_2^2 \xi_2^2)/U_\tau^2}\right) \exp\left(-\sqrt{\sqrt{(\beta_1^2 \xi_1^2 + \beta_2^2 \xi_2^2)} U_\tau/f\delta^2}\right), \quad (2)$$

where  $f = \omega/2\pi$ ,  $\nu$  is the kinematical viscosity,  $\alpha_1 = 0.43$ ,  $\alpha_2 = 2.98$ ,  $\beta_1 = 0.25$ , and  $\beta_2 = 5.53$ .

The coherence model (2) greatly differs from all models offered before. It completely denies Corcos's similarity on all frequencies including high ones. The features of model (2) are best for considering with the help of a streamwise coherence scale  $A_1$  which is entered by a ratio

$$|\gamma(\xi_1, 0, \omega)| = \exp(-|\xi_1|/A_1). \quad (3)$$

In the Corcos's model the streamwise coherence scale is equal

$$A_{1c} = U_c/(\alpha_1 \omega). \quad (4)$$

In the model of Ref. [1] such a scale may be defined as

$$A_1 = \delta \left[ \frac{\alpha_1}{(2\pi)^{3/2}} \left(\frac{\nu}{U_\tau \delta}\right)^{1/2} \left(\frac{\omega \delta}{U_\tau}\right)^{3/2} + \left(2\pi \beta_1 \frac{\delta}{\xi_1} \frac{U_\tau}{\omega \delta}\right)^{1/2} \right]^{-1}. \quad (5)$$

In Fig. 2 scales (4) and (5) versus dimensionless frequency are shown. To the Corcos's similarity there corresponds recession of scale  $A_1$  as  $1/\omega$ . For the model of Ref. [1] the recessions at high frequencies are proportional to  $\omega^{-3/2}$ . It contradicts all known experimental [3–7] and theoretical [8,9] results which testify that the Corcos's similarity is retained at high frequencies. In accordance with Refs. [3–7] these frequencies satisfy the inequality  $\omega\delta/U_\tau > 50$ –100. At low frequencies  $\omega\delta/U_\tau < 50$ –100 the Corcos's similarity does not hold because coherence length scales cannot grow limitless as predicted by the Corcos's model (dotted line in Fig. 2) because the finite thickness of a boundary layer limits the size of length scale. With this assertion the authors [1] also agree, when they completely fairly declare, that "...for normalized frequencies lower than 100 ... the boundary

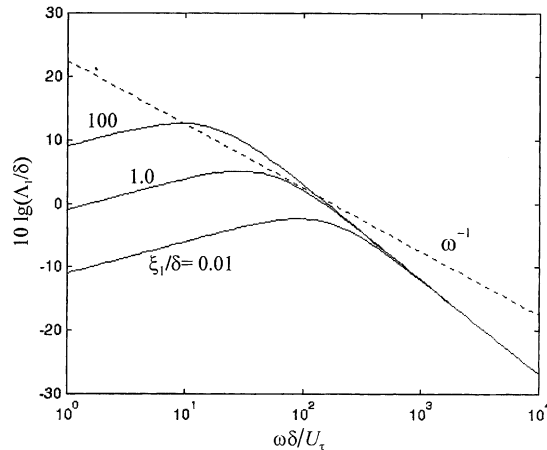


Fig. 2. Streamwise coherence length scales of wall pressure fluctuations dotted line—Corcos's model for  $\alpha_1 = 0.124$ ,  $U_c = 0.8U$  and  $U = 40$  m/s. Solid lines—model presented in Ref. [1] for  $U = 40$  m/s, and  $U_\tau = 1.47$  m/s.

layer thickness determines the largest possible scales” (p. 491). However, this statement contradicts model (2) according to which streamwise coherence length scales (5) in the intervals  $\omega\delta/U_\tau = 1\text{--}100$  grows limitless when the separation  $\xi_1$  increase (solid lines in Fig. 2). The features of transversal coherence scales are quite analogous.

It is necessary to notice that for many problems of flow-induced sound and vibration the essential information is a wavenumber–frequency spectrum of pressure fluctuation at very small wavenumber  $k \ll \omega/U_c$  [2,8–13]. The production of such information by cross-spectra measurements only is practically impossible because the wavenumbers  $k \ll \omega/U_c$  correspond to very large space intervals  $\xi_i/\delta$  at which the coherence function is considerably small. It is impossible to obtain the wavenumber–frequency spectrum by the Fourier-transformation of model (2) by analytical methods, so it should be obtained numerically. Unfortunately, Ref. [1] does not contain such information.

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